Exact Solutions of the *N*-dimensional Radial Schrödinger Equation with the Coulomb Potential via the Laplace Tranform

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The second-order *N*-dimensional Schrödinger differential equation with the Coulomb potential is reduced to a first-order differential equation by means of the Laplace transform and the exact bound state solutions are obtained. It is shown that this method solving the Schrödinger equation may serve as a substitute for the factorization approach also in lower dimensions. — PACS numbers: 03.65.Ge.

Key words: Bound State; Coulomb Potential; Laplace Transforms.

With the motion of a particle in an *N*-dimensional Euclidian space, the time independent radial Schrödinger equation for any integral dimension is shown below [1 – 7]

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{N-1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-\frac{L(L+N-2)}{r^2} -\frac{2m}{\hbar^2}V(r)+\frac{2m}{\hbar^2}E\right]R_L(r)=0. \qquad (1)$$

For the Coulomb potential $V(r) = -Ze^2/r$ Eq. (1) turns into

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{N-1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-\frac{L(L+N-2)}{r^2} +\frac{2mZe^2}{\hbar^2 r}+\frac{2m}{\hbar^2}E\right]R_L(r)=0. \qquad (2)$$

If one introduces

$$2mZe^2$$